Optimal Partial Tiling of Manhattan Polyominoes

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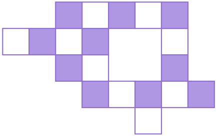
1. CONTEXT : domino tiling

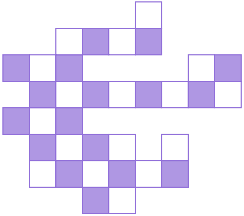
Question: Can a given polyomino P be completely recovered with non-overlapping dominoes?



Polyomino

$O(n \log^3 n)$

(Bodini and Fernique, 2006)



Polyomino without holes
(simply connected)

$O(n)$

(Thurston, 1990)

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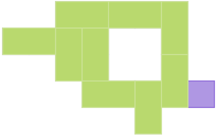

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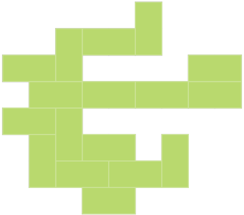

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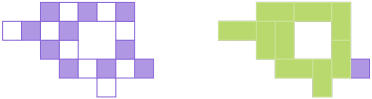
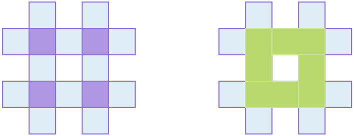
2. PROBLEM

Existing algorithms say "yes" or "no"
(+ give the tiling *when* the answer is "yes").

When answer is "no", no additional information:



1 square not covered?

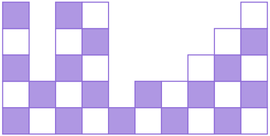


HALF the polyomino??

New question: what is the fewest number of squares that have to be left uncovered (when still tiling with non-overlapping dominoes)?

3. OUR WORK

Definition (Manhattan polyomino).
Adjacent columns which all *shoot out* from the same base line.



Entirely described by the sequence of heights [of the columns]:

```
5,2,5,5,1,2,2,3,4,5
```

Definition (black/white-dominant column).
Oddly-sized columns ["odd" as in $2k+1$] with more black/white squares.

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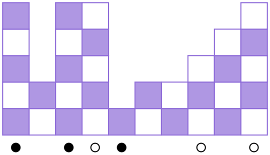

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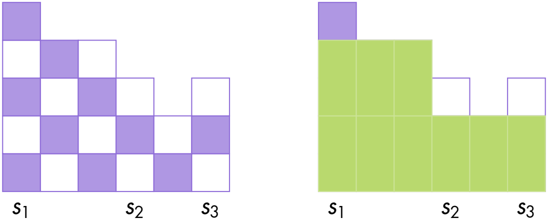
A couple facts to guide intuition

Fact 1. The oddly-sized columns *are* the problem.

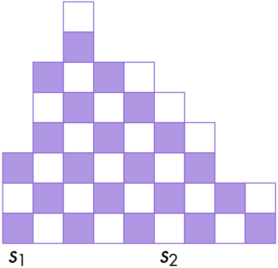


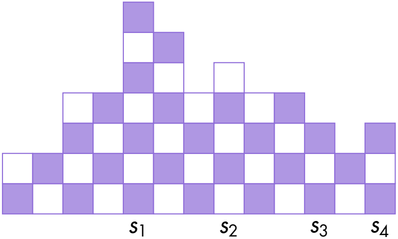
Fact 2. Each domino covers one dark and one light square.

Optimal partial tiling of Manhattan polyomino = pairing as many white-dominant columns with black-dominant columns.

Our planing transformation

Two “adjacent” oddly-sized columns of different colors...





Planing transformation = templates to “*even out*” oddly-sized columns.

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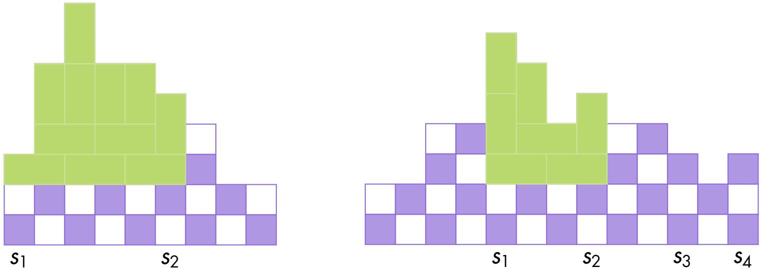

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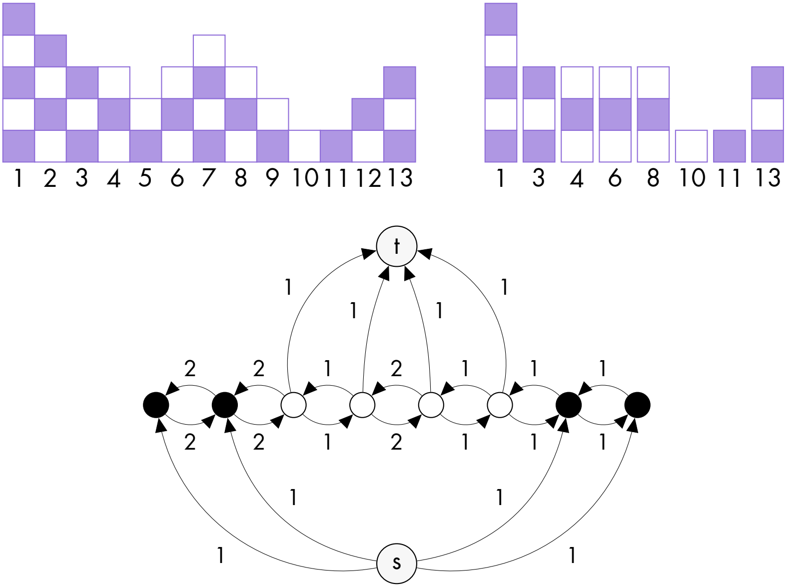
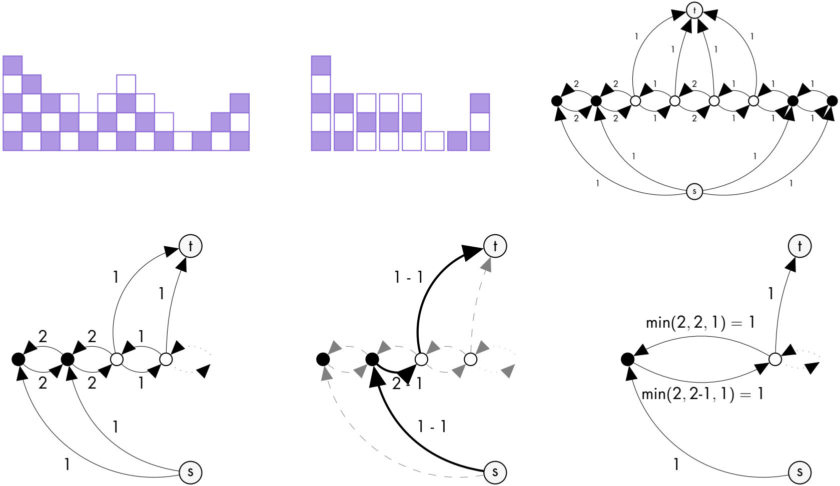
The flow network construction



Saturating paths



Saturating one path in the flow ⇔ applying one planing transformation

4. THE ALGORITHM

… is given in the poster.

- greedy application of the planing transformation;
- linear in the size of the input.